EPSRC
Pioneering research and skills

Time-dependent stochastic modelling for Emergency Medical Services in Wales

How should we flex the workforce to match fluctuating demands?

Theory LANCS Practice

hmc² Health Modelling Centre Cymru

Dr Julie Vile, Prof. Paul Harper, Dr Jonathan Gillard & Dr Vincent Knight

Cardiff School of Mathematics, {VileJL, Harper, GillardJW, KnightVA}@Cardiff.ac.uk

Cardiff University / Prifysgol Caerdydd

1 The problem

❖ Akin to most Emergency Medical Services (EMS), the Welsh Ambulance Service Trust (WAST) is under increasing pressure to improve performance to meet Government set response time targets.

❖ My research develops advanced analytical techniques, culminating in a suite of decision tools to aid the Trust (i) **predict future demands**; (ii) **evaluate fleet size** to meet government targets; and (iii) **enable efficient scheduling** of crews.

Figure 1: MATHEMATICAL MODELS TRACK PATIENT FLOWS THROUGH THE SYSTEM

DEMAND FORECASTS ARE GENERATED (Find arrival rates of high (H) & low priority (L) calls, λ_H & λ_L, by hour of day)

λ_H → Calls are triaged, forming a priority queue ← λ_L

NPRT Queue

SERVICE TIMES ARE COMPUTED FOR FIRST RESPONSES AND AMBULANCES

μ

(Find average time, μ, required to reach, treat and transport patients)

TIME-DEPENDENT AND PRIORITY QUEUEING THEORY TECHNIQUES ARE APPLIED TO DETERMINE VEHICLE AND STAFF REQUIREMENTS

2 My approach

(i) Improve demand forecasts: SSA

❖ Prior planning models generally assume demand is known as a precursor or base estimates on smoothing models e.g. ARIMA, that require restrictive assumptions.

❖ My research considers the ability of a novel non-parametric technique known as SSA to account for the heavily time-dependent nature of demand (see Fig. 2).

Figure 2: DAILY DEMAND PROFILE

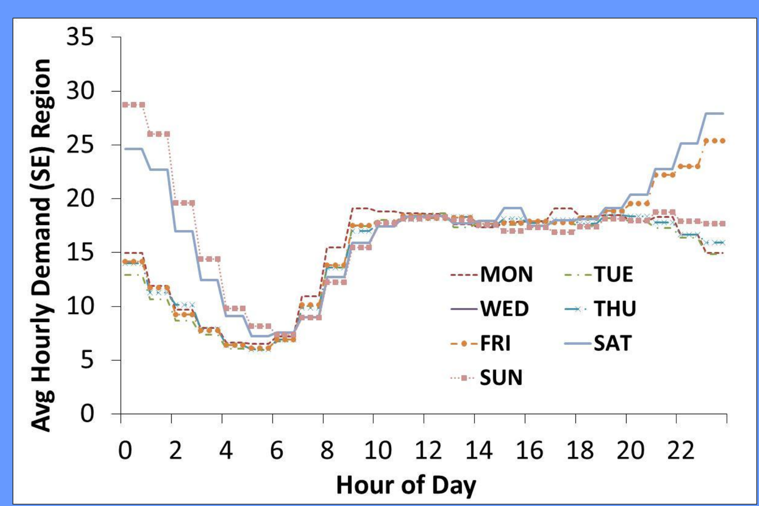
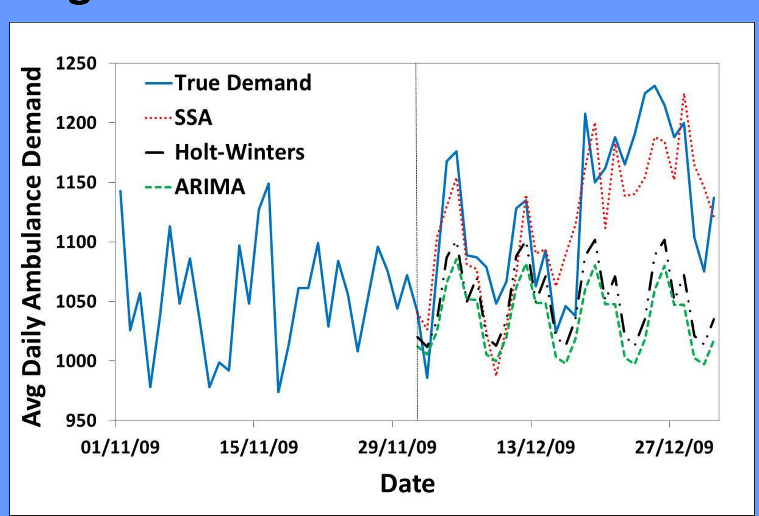


Figure 3: SSA MATCHES DEMAND



❖ SSA requires no assumptions, other than potential structure to the data. It anticipates peaks and troughs in demands in a two-step process:
(1) Decomposition: The time series is decomposed into a sum of the trend, periodic components and noise.
(2) Embedding: Specific components are selected to reconstruct the series.

Figs. 3 and 4 show that SSA forecasts outperform traditional techniques.

Figure 4: FORECASTS ACCURACIES

Average MAPE	SSA	Optimal ARIMA	Holt-Winters
Retrospective	2.05%	4.35%	3.96%
1-Day Forecast	3.40% (0.023)	3.45% (0.028)	2.98% (0.025)
7-Day Forecast	3.90% (0.011)	5.31% (0.023)	4.02% (0.024)
14-Day Forecast	3.64% (0.006)	6.16% (0.008)	4.57% (0.021)
21-Day Forecast	3.54% (0.003)	6.67%	4.78% (0.005)
1-Month Forecast	3.25%	7.63%	6.18%

(ii) Compute staffing requirements

❖ **The next step involves converting the demand forecasts into optimum staffing levels** that satisfy the Government response time targets.

❖ Service quality is measured by the probability of an excessive wait. My research adjusts the standard waiting time formula $W_q(t)$ to evaluate the likelihood of high & low priority patients subjected to long response times.

$$P(W_q(t) > x) = \sum_{n=s_0}^{+\infty} P(W_q^n(t) > x) p_n(t) \text{ where } P(W_q^n(t) > x) = \sum_{i=0}^{n-s_0} \frac{a^i e^{-a}}{i!} \text{ if } n \geq s_0,$$

$a = \mu s_0 x; p_n(t)$ prob(n paitents in system); s_0 = no. of ambulances deployed over $[t, t+x]$

$$P(W_q H(t) > x_H) = \sum_{n=s_0}^{+\infty} P(W_q^n(t) > x_H) p_{H_n}(t)$$

where $p_{H_n}(t)$ = prob (n patients in system, excduing low priority patients in the queue)

$$P(W_q L(t) > x_L) = \sum_{n=s_0}^{+\infty} P(W_q^n L(t) > x_L) p_n(t)$$

where $P(W_q^n L(t) > x_L) = P(j\ H's \text{ arrive in } x_B) \sum_{i=0}^{n-s_0+j} \frac{a^i e^{-a}}{i!}$ if $n \geq s_0$

❖ By embedding the developed formulae within numerical (Euler) methodology and iteratively computing the measures for different staffing teams, optimum hourly staffing levels can be generated.

❖ An approximate method (SIPP) is shown capable of producing rough recommendations quickly.

(iii) Optimise the staff roster

❖ Simulated Annealing heuristics are used to facilitate the **generation of inexpensive, high quality staff rosters.**

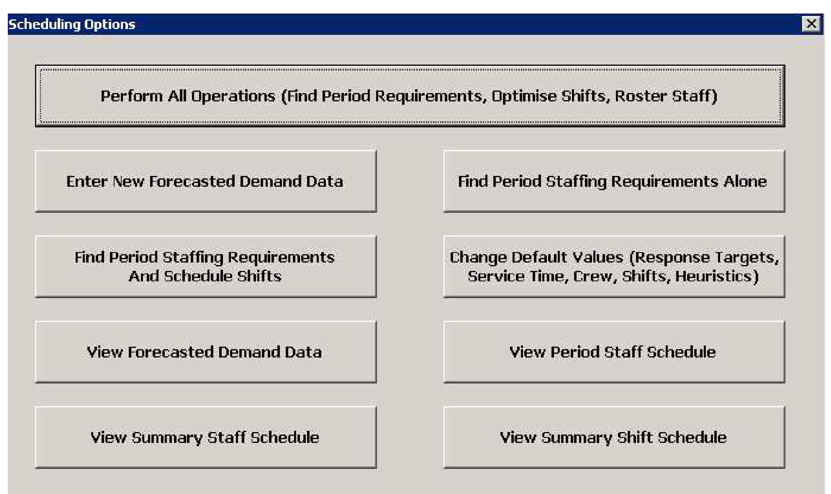
3 The solution

❖ The methods are finally amalgamated in an Excel-based planning tool, with a user friendly interface that can be used by WAST employees to forecast demand, make capacity planning decisions and develop rosters independently.

❖ A hybrid method is programmed within it which considers both numerical and approximate techniques to produce minimum coverage requirements.

❖ The generic nature of the programmed techniques means **the tool could be adopted by any ambulance service internationally.**

Figure 4: FRONT-END INTERFACE



❖ The unique linking together of the methods in a tool which captures time-dependency and two priority classes enables this research to considerably outperform previous research works.

4 Impact & publications

"The work is an extremely relevant contribution to implementing policy and procedural changes at WAST"
- Clinical R&D Manager, WAST

❖ Vile, J.L. (2013), Time-dependent stochastic modelling for predicting demand and scheduling of emergency medical services, Ph.D Thesis, Cardiff University. U.K.
❖ Vile, J.L., Gillard, J.W., Harper, P.R., and Knight, V.A. (2012). Predicting ambulance demand using singular spectrum analysis, Journal of the Operational Research Society, 63(11) pp. 1556-1565
❖ Vile, J.L., Gillard, J.W., Harper, P.R., and Knight, V.A. (Under review.) Incorporating the effects of workforce adjustments at shift boundaries in time-dependent and priority service systems.