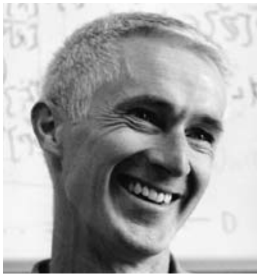
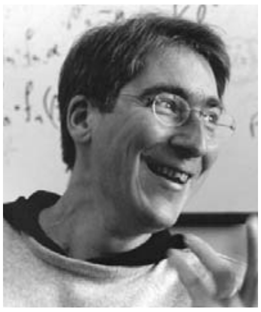
MATHEMATICS TODAY



Ken Brown
Professor of Mathematics,
University of Glasgow
and Vice-President of the London Mathematical Society



Paul Glendinning
Professor of Applied Mathematics,
University of Manchester
and Vice-President of the Institute of Mathematics and Its Applications

Faces of Mathematics.
Photographer: Marc Atkins

British mathematics has a stunning history, spanning at least 400 years. To understand the motion of the planets, Isaac Newton developed the mathematical tools that are still used to describe the motion of almost anything.

In the 1860s James Clerk Maxwell wrote down the relativistic equations of light and radio waves, anticipating aspects of Einstein's theory by twenty years. R A Fisher developed the mathematical theory of statistics in its modern form almost single-handedly in the 1920s whilst working at the Rothamsted Experimental Station; and Alan Turing used mathematics to decode the German enigma machines in the 1940s, developing the first computers in the process. In 1994, more than three hundred and fifty years after the problem was first posed, Andrew Wiles proved Fermat's Last Theorem; Wiles will return from the US to a post at Oxford later this year.

To assess the state of current mathematical science, the Engineering and Physical Sciences Research Council (EPSRC) commissions regular reports from international experts. The International Review of Mathematical Sciences 2010 (IRMS 2010)[1], was published this Easter, the first report since 2004. Its conclusion is that 'UK mathematical sciences research is world-leading in some fields, outstanding in many others and strong overall'. The Executive Summary (p iv) adds

"Two major factors that contribute to the present excellence of the UK academic mathematical sciences enterprise are its diversity – in area, group size and size of institution – and its geographically distributed nature."

The report goes on to examine both the activity and the processes involved in mathematical research in the UK. Its publication provides an excellent opportunity for the UK mathematics community to reflect on recent achievements and frame future prospects.

Publication of the IRMS report coincides with far-reaching and controversial changes in research funding policy signalled by the EPSRC's 'Shaping Capability' agenda[2]. The Research Council intends to take a more proactive role in commissioning and sponsoring research, identifying research areas for growth and special support, rather than simply supporting excellence as advised by academic and industrial experts. In what follows we shall try to explain how and why the mathematical sciences must exercise central roles in the culture and the economy of any successful modern society; and we shall also aim to show why EPSRC's current strategy risks making these roles unsustainable for UK mathematical science.

It is hard to overstate the importance and the ubiquity of mathematics. The IRMS 2010 expresses it well (again from the Executive Summary),

"the mathematical sciences provide a universal language for expressing abstractions in science, engineering, industry and medicine; mathematical ideas, even the most theoretical, can be useful or enlightening in unexpected ways, sometimes several decades after their appearance; the mathematical sciences play a central role in solving problems from every imaginable application domain; and, because of the unity of the mathematical sciences, advances in every sub-area enrich the entire field."

However, mathematical science is also a hugely important discipline in its own right, with its own culture and intellectual imperatives, its own history over millennia, and its own 'Grand Challenges'. It is important to see mathematics in its entirety and not be distracted by the crude and misleading distinction between theory and applications, often expressed as 'pure' versus 'applied' mathematics.

Misled by its daily usefulness, we might see mathematical science as a stagnant well of techniques from which one can ladle out exactly what is needed to deal with a given problem. This is far from the truth. The "right" mathematics may be languishing in obscurity, having been developed many years earlier; or it may be in a field with no apparent connection with the matter at hand; or it may well not yet have been discovered.

Many examples can be given of each of these cases. For example, the medical imaging techniques used every day in every hospital in the land depend crucially on abstract mathematical analysis of the early 20th century; and the "matrix formulation of quantum mechanics", developed in the late 1920s, hinged on the then very obscure – but now school-level – matrix algebra, studied by English algebraists 60 years earlier. Of course quantum mechanics itself was in the 1920s regarded as completely useless, but now underpins our digital universe. A problem we still do not know how to approach is that of extracting the important information hidden in huge data sets. This is one of the key challenges for genomic biology, and statisticians are currently making important advances in developing new methodology to address it.

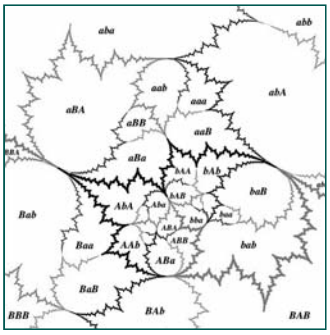
The well and ladle metaphor is grossly misleading in a second way: it wrongly suggests that those working on applications don't themselves produce fundamental mathematics. In fact the reverse is the case. Newton's discovery of the calculus is of course the first and greatest of many British examples. This age-old interchange between mathematical science and physics continues undiminished today: stemming from the pioneering work of Sir Michael Atiyah and his students, the UK has been a world leader for 50 years in the convergence of parts of physics with the 'purest' reaches of algebra, geometry and topology.

Mathematics doesn't just solve problems it provides insights which can lead to more far-reaching advances. In the 1990s Frank Kelly (University of Cambridge), worked on BT's new routing architecture where a major issue is how to deal with blockages in the network. The natural 'technological' solution is to have full knowledge of the state of the system and compute the most efficient route from the blocked point. Kelly showed that the far simpler and more robust method of sending the call to a nearby node at random and then taking the standard route from there was almost as efficient, far cheaper to implement, and far less likely to malfunction. This insight, that a less sophisticated, simple solution can be almost as good and far cheaper than a technically perfect solution, is now a recognised design feature of networks.

The intricate interdependence of fundamental mathematical science and application makes it very hard to steer mathematical research in any meaningful way. This doesn't mean that we shouldn't try, but it does suggest that the best compass to use may be one which seeks out the highest quality and the most promising directions, in each particular field. Mathematicians and statisticians should continue to put huge efforts into seeking solutions to society's challenges, but the health of the core discipline is a vital feature of a country's mathematical research framework.

It could be argued that given the severity of the current economic crisis we should leave fundamental research to other countries and focus on the applications. This is to misunderstand the nature of mathematical research: core and applied mathematics are inextricably interlinked and, as we've tried to show, to spot the right mathematics for a given application requires immersion in the well and can't simply be done by wielding a ladle from above. Moreover, it often happens that deep understanding of the mathematical science actually generates the application. A famous example is the Pagerank algorithm at the heart of Google, which relies on the same matrix algebra that was crucial for quantum mechanics.

How do people actually *do*

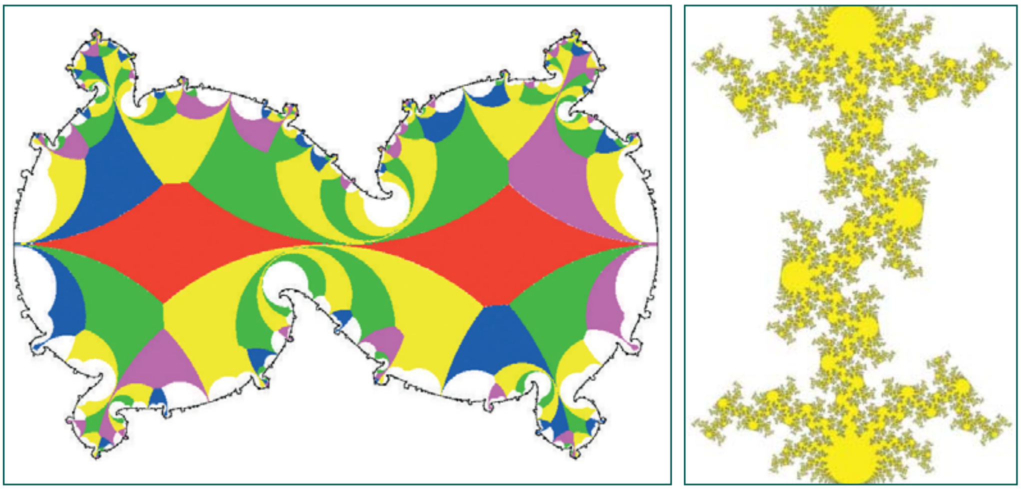


research in mathematics? The answer, typically, is: by reading a bit, perhaps talking to colleagues and students (both down the corridor and across the planet), and by thinking a lot. Consequently, the working research mathematician's requirements are relatively few – good internet access, a quiet and warm place to work, and plenty of time and coffee! Except in some cases involving large interdisciplinary activity, what she or he *doesn't* usually need is a big team working on the same problems in the same place, or expensive equipment. These factors make it easy to achieve the diversity and geographic distribution highlighted as virtues by the IRMS 2010. They also ensure that UK mathematical scientists are well positioned, in terms both of geography and subject coverage, for the absolutely crucial task of teaching undergraduate and postgraduate students.

In both teaching and research UK mathematics is a large activity: 1129 international-level mathematical scientists (FTE) were submitted to the last research assessment exercise compared with 729 in physics and 957 in chemistry[3]. The dual funding (Funding Council/ RCUK) support for UK universities means that research and teaching are linked, so that students taking degrees in mathematical science have the opportunity to see the subject as the living, developing discipline that it is. And it's an opportunity which more and more students are taking: there were 5475 graduating students in mathematical science in the UK in 2007-8, almost as many as chemistry (2965) and physics (2765) combined. The figure for graduate mathematicians has been steadily rising for over a decade now: in 2000 it was 3500[4].

Mathematics graduates are employed in banking, medicine, pharmaceuticals, manufacturing, communications and other advanced technology, teaching, government departments, actuarial and accountancy as well as going into business for themselves. Mathematics is rightly seen as a challenging degree by employers and valued for its transferable skills; indeed postgraduate mathematical scientists have the highest average starting salary among all UK holders of postgraduate degrees[5]. This crucial contribution to the country's economy is only possible by virtue of the wide distribution of research excellence which ensures that mathematics research and teaching is accessible throughout the country.

So much for the scale of the enterprise, but what about diversity and quality? Given the size of the mathematics research community it is not surprising that most areas of the discipline are represented within the UK. British-based mathematical scientists are pioneering world-class work in fields as diverse as models of cancer growth and properties of sequences of prime numbers. They are involved in applications ranging from the analysis of option pricing to the assessment of medical procedures. And their excellence is recognised up to the very highest level: six UK mathematicians hold Fields Medals[6]. This is officially known



With minimal input, these intricately beautiful computer generated fractal graphics are made by repeating simple geometrical operations many times. Behind them lie deep mathematical discoveries of recent years. Reproduced with permission from Indra's Pearls, copyright Cambridge University Press.

as the International Medal for Outstanding Discoveries in Mathematics, but unofficially as the 'Nobel Prize for mathematics'. Two or three of these are awarded once every four years, for work done before the age of forty.

British mathematical science wins financial backing not only from the HE Funding Councils and from the Research Councils, but from European agencies, from charities, from government and industry, and from many outside bodies. To give just one recent and very notable example: the Oxford Centre for Collaborative Applied Mathematics[7] (OCCAM) has been created with £20m backing from the King Abdullah University of Science and Technology (Saudi Arabia).

Inevitably there is room for improvement. The IRMS 2010 criticises the UK for the poor representation of women in mathematics, and also points out that the brevity of UK doctoral training compared with mainland Europe and the US can put young UK mathematicians at a disadvantage compared to their international peers. Both these are points the community and the universities are addressing, but, particularly in the case of women in mathematics, there is some way to go.

Despite the best efforts of people such as Marcus du Sautoy and Ian Stewart, we could do better in telling the public about the excitement and applicability of mathematics – sites such as the excellent Mathematics Matters[8] of the Institute of Mathematics and its Applications point the way here.

Although mathematics is relatively cheap compared with experimental science, 'cheap' does not mean 'free' – money is badly needed to maintain and widen the pipeline for fresh talent, from PhD training through to postdoctoral fellowships and beyond. Research grant support for established mathematical scientists gives them essential opportunities to interact on a global stage with their peers, and provides vital periods of uninterrupted time for research.

A crucial and more subtle point about research council support is often missed – namely, there are unintended negative consequences of low and reducing levels of funding, beyond the straightforward loss of support for current research. University administrations, under pressure to maximise external funding, are increasingly reluctant to make new appointments in fields where research council support is low, so that, over time, the geographic and subject diversity highlighted above will be threatened.

We share the widespread fear that the future of UK mathematical sciences is under threat. The research grant commitment of the EPSRC Mathematics Programme has been in decline since 2007-8, at a time when funding for other disciplines in EPSRC's portfolio was still increasing. At a modest £12m, it was the same in cash terms in 2009-10 as it had been in 2003-4. Over this same period the total EPSRC research grant commitment increased from £378m to £459m, the latter figure including £88m for physical sciences and £72m for ICT[9]. It is in this already very challenging landscape that the EPSRC is now rushing through its ill-considered 'shaping capability' agenda.

This agenda is being implemented before the mathematics community has been properly consulted. It places strategic decisions in the hands of administrators, with priorities such as the centralisation of research that do not necessarily fit the mathematics landscape. There is a real danger that the geographically distributed excellence in UK mathematical science, developed over many years with the support of HEFCE and the research councils, is about to be seriously diminished.

Acknowledgements: We are grateful to colleagues on the Council for Mathematical Sciences for comments and suggestions that have been incorporated into this article.

1 The IRMS 2010 Report can be downloaded at www.epsrc.ac.uk/newsevents/pubs/corporate/intrevs/2010maths/

2 Details of the EPSRC Shaping Capability Policy are at www.epsrc.ac.uk/plans/implementingdeliveryplan/goals/shapingcapability/
Council of Mathematical Sciences (CMS) responses are at www.cms.ac.uk/submissions.html

3 Figures obtained by multiplying the percentage of outputs in quality bands 3* and 4* by the number of academic staff in each RAE submission, and summing over all submissions.

4 Figures from HESA; see www.iop.org

5 Adrian Smith's report One Step Beyond: Making the most of postgraduate education, p. 94 (March 2010)

6 Atiyah, Baker, Borcherds (based in the US), Donaldson, Gowers and Roth.

7 www.maths.ox.ac.uk/groups/occam

8 www.ima.org.uk/i_love_maths/mathematics_matters.cfm

9 Taken from Table 3 of CMS submission to House of Commons Select Committee inquiry on the Spending Review 2010, www.cms.ac.uk/submissions.html
Assembled from tables in EPSRC Annual Reports